

where ρ : density of the plate material; h : plate thickness; D : flexural rigidity of the plate and μ : Poisson's ratio. Poisson's ratio is taken equal to 0.30 for present calculations.

Approximating W by means of

$$W \simeq W_\alpha = \sum_{j=0}^3 A_j \left[\alpha_j \left(\frac{r}{a} \right)^\gamma + \beta_j \left(\frac{r}{a} \right)^2 + 1 \right] \left(\frac{r}{a} \right)^{2j} \quad (9)$$

where the α_j 's and β_j 's are such that each coordinate function contained in (9) satisfies identically the boundary conditions (8b) and (8c). Applying Galerkin's procedure one obtains, minimizing with respect to γ , that the first, nonzero, eigenvalue is $\sqrt{\rho h/D\omega_1 a^2} = 9.003$ which coincides with the exact value, within 4 significant figures. The higher eigenvalues are, again, obtained minimizing the higher roots of the frequency equation with respect to γ [5].

It is important to point out that this optimization procedure has also been implemented in finite element codes [8].

REFERENCES

- [1] P. Sh. Fridberg and I. M. Yakover, "A procedure for defining behavior of weight functions near the edge for best convergence using the Galerkin method," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1661-1667, Aug. 1992.
- [2] R. Schmidt, "Technique for estimating natural frequencies," *ASCE J. Eng. Mechanics*, vol. 109, pp. 654-657, 1983.
- [3] C. W. Bert, "Use of symmetry in applying the Rayleigh-Schmidt method to static and free vibration problems," *Industrial Math.*, vol. 34, pp. 65-67, 1984.
- [4] P. A. A. Laura, L. C. Nava, P. A. Laura, and V. H. Cortinez, "A modification of the Galerkin method and the solution of Helmholtz equation in regions of complicated boundary shape," *J. Acoust. Soc. America*, vol. 77, pp. 1960-1962, 1985.
- [5] P. A. A. Laura and V. H. Cortinez, "Optimization of eigenvalues when using the Galerkin method," *J. American Inst. Chem. Engineers*, vol. 32, pp. 1025-1026, 1986.
- [6] —, "Optimization of the Kantorovich method when solving eigenvalue problems," *J. Sound Vib.*, vol. 122, pp. 396-398, 1988.
- [7] C. W. Bert, "Application of a version of the Rayleigh technique to problems of bars, beams, columns, membranes and plates," *J. Sound Vib.*, vol. 119, pp. 317-326, 1987.
- [8] P. A. A. Laura, J. C. Utjes, and G. Sánchez Sarmiento, "Nonlinear optimization of the shape functions when applying the finite element method to vibration problems," *J. Sound Vib.*, vol. 111, no. 2, pp. 219-228, 1986.

Corrections to "A New Formulation of the Boundary Condition at Infinity for a Hybrid Radiation Mode and Its Application to the Analysis of the Radiation Modes of Microstrip Lines"

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In the above paper [1] a few corrections should be introduced as a result of additional analysis and numerical calculation:

- For the odd case ($J_z(\alpha)$ - odd) application of (12) leads to infinite power flux for both solutions of (11). The analysis of behavior of $A_E^{(2)}(\alpha)$ at the points $\alpha = 0$ and $\alpha = \gamma_2$ yields the value of the amplitude which gives the finite power flux of perturbed LSM mode. The proper choice of this amplitude is:

$$A_E^{(2)}(\alpha) \sim \gamma_2^{p+1/2} \quad (1)$$

where $p > 0$. The perturbed LSE odd solution of (11) shows infinite power flux—this mode has no physical meaning and should be neglected.

- The iterative procedure described in Section III of [1] is divergent for even case ($J_z(\alpha)$ - even). We can, however, rearrange (11) to an alternative set of equations:

$$A_E^{(2)}(\alpha) = f_3[A_E^{(2)}(\alpha), A_H^{(2)}(\alpha)] \quad (2)$$

$$A_E^{(2)}(\alpha) = f_4[A_E^{(2)}(\alpha), A_H^{(2)}(\alpha)] \quad (3)$$

Spectral amplitude $A_H^{(2)}(\alpha)$ is treated now as a known function and $A_E^{(2)}(\alpha)$ is found by the same iterative procedure (in Fig. 3 [1] we should only replace $A_H^{(2)}(\alpha)$ with $A_E^{(2)}(\alpha)$). The analysis of behaviour of $A_H^{(2)}(\alpha)$ at the points $\alpha = 0$ and $\alpha = \gamma_2$ yields the amplitude which results in the finite power flux of perturbed LSE mode. The proper choice of this amplitude is:

$$A_E^{(2)}(\alpha) \sim \gamma_2^{p+1/2} \quad (4)$$

where $p > 0$. The second solution (i.e. perturbed LSM even mode) should be neglected as a mode showing infinite power flux. Numerical results of convergence of the proposed procedure are shown in Table I.

In effect we conclude that the hr mode of microstrip line can be treated as a superposition of perturbed LSM odd and LSE even modes. Numerical calculation (see Table I) showed that the modifications did not change the fast convergence of iterative procedure.

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TABLE I
 NUMBER OF ITERATIONS REQUIRED FOR CONVERGENCE FOR LSE EVEN AND
 LSM ODD PERTURBED H_R MODES OF MICROSTRIP LINE [1]
 $(w = D = 1 \text{ mm}, \epsilon_{r1} = 9.6)$, COMPUTATIONS WERE CARRIED OUT
 AT FREQUENCY 15 GHz WITH RELATIVE ERROR $\eta = 0.01$

β [rad/mm]	0.3	0.2	0.1	0.01	-j0.1	-j0.2	-j0.3
odd case	3	5	3	3	3	3	3
even case	3	4	2	2	2	3	2

REFERENCES

[1] W. Zieniutycz, "A new formulation of the boundary condition at infinity for hybrid radiation mode and its application to the analysis of radiation modes of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1294-1299, Sept. 1990.